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Vertical Earth Coupling Kelvin Line Theory
Technical Bulletin 2007-13

GENERAL	NOTES
This technical bulletin provides the basic theory behind the vertical Earth coupling of a ground source heat pump. The bulletin computes a sample problem of a vertical application for hybrid and simple recirculation. The bulletin also provides tables for the recommended design of recirculating well systems (closed loop, open pipe). This bulletin also discusses the use of hybrid ground water/ground source recirculation systems. This technical bulletin does not replace engineering design and is intended only to teach and familiarize the technical reader with design methods. **STANDING COLUMN WELL** The Standing Column Well system has been used by Water Energy since 1979 with good success; particularly with applications that do not produce sufficient water for overflow recirculation methods. This Standing Column Well application, has ground from the requirement of larger and larger commercial/industrial ground source heat pump applications. In these applications, overflowing of large amounts of water, or achieving high production wells may not be possible. A typical Standing Column Well system is as shown in figure 1. Note the Earth coupling is via water at atmospheric pressures and does not employ any anti-freeze solutions with their attendant environmental problems. Se our tech bulletin #38 for further discussion on the use of anti-freeze.	

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First Published in September 1991 -

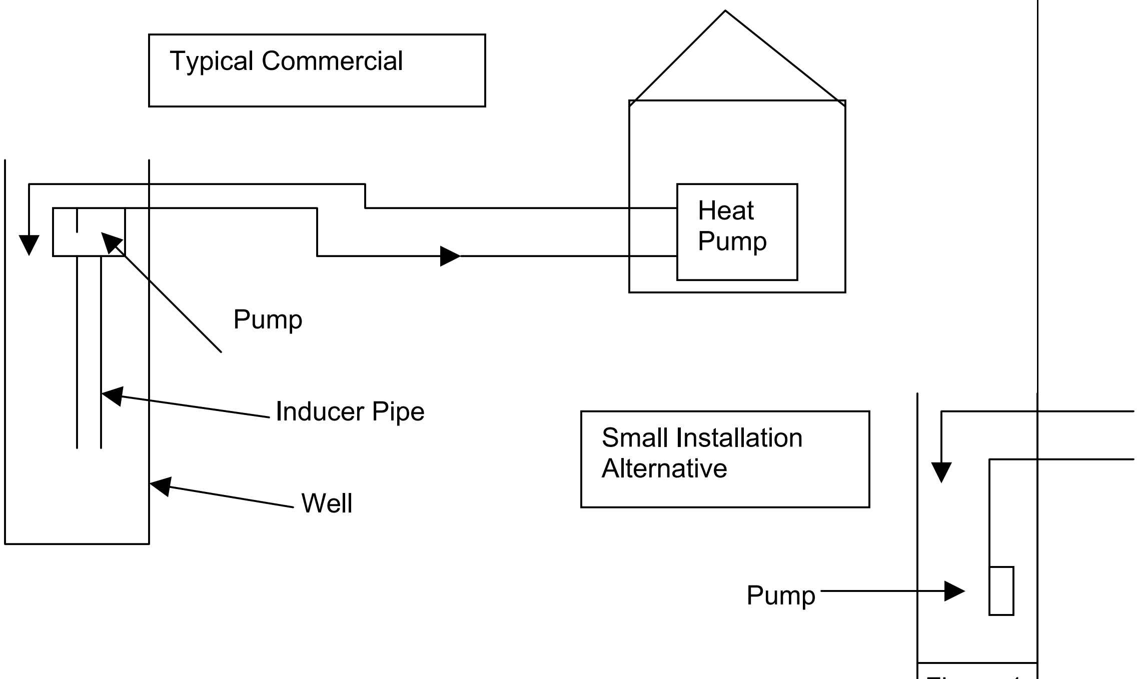

Figure 1

In this application, the water is taken from the bottom of the well by an “induce dip tube,” attached to a submersible pump. Methods of inducer tube attachment and support is usually the property of the well/pump contractor and is not covered in this bulletin.

*Note: the water is always returned below the static level of the well. In this manner, well scaling is inhabited as the naturally occurring dissolved carbon dioxide in underground water is maintained in the solution since the atmospheric pressure on the solution is maintained.

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THEORY

The recirculation theory is developed from material from Oklahoma State University (Dr. Jim Bose, 1984), "Heat Pump Applications," (Kemler & Oglesby, McGraw-Hill 1950), "Closed Loop/Ground Source Heat Pump Guide," (IGSHPA, 1988), and our own field observations over a ten year period.
The heat transfer performance of a vertical well can be described by the "Kelvin Line Theorem." The theorem was developed by Ingersoll, "Heat Conductance with Engineering and Geological Applications" (McGraw-Hill, 1948). The theorem can be stated by:

$$T-T_0 = (Q'/(2\pi k_s)) * \int (e^{(-\beta^2)} / \beta)d\beta$$

Integrated from x to infinity, where

$$x = r / (2\sqrt{\alpha t})$$

Where:

- T = soil temperature at a distance from the well (°F)
- T_0 = Initial soil temperature (°F)
- Q' = Heat transferred from well column (BTU/hr/Lft)
- r = Radial distance from line source (ft)
- k_s = Thermal conductivity of the soil (BTU/ft* °F)
- α = Thermal diffusivity ($k_s/\rho c_p$) (sqft/hr)
- ρ = Soil density (lb/cu ft)
- c_p = Soil specific heat (BTU/lb * °F)
- t = Heat pump running time (hr)
- β = Variable of integration
- I(x) = value of integral (Tables used)

As can be noted, these variables appear to have constraints.

The temperature can only be reduced to about 35°F, the radial distance from the line source can reasonably be 50 feet, the soil density cannot change, neither soil conductivity nor diffusivity can change….or so it appears.

HYBRID WELL-RECIRCULATION GROUND COUPLING

Our observations have indicated a small amount of well water overflow can make the conductivity appear to increase by factors of 2.5 or better. Since water transports through the soil at a faster rate than the heat moves through the soil, the effective thermal conductivity appears to be much higher, with the equivalent heat transport faster. (Data by MACE Engineers/Architects, and others 1979-88).

In this manner, 80-90% of the water is returned to a properly sized well and the balance of the water is returned to the Earth some distance away. This distance is conservatively defined by:

$$D = \sqrt{(.2 * Q)}$$

Where:

- D = Distance to a second recharge well (feet)
- Q = Design heat loss (BTU/hr)

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This formula is based upon information from the National Well Water Association and applies to an average residential design. The use of a second recharge well is not always required, and this information is included for completeness. See our tech bulletin #25.

Figure 3 is a listing of typical Earth heat transfer coefficients.

Figure 3 – Typical Earth Coefficients

		k_s	ρ	c_p
Rock	Dense	2.0	200	0.20
	Average	1.4	175	0.20
Soil	Heavy Damp	.75	131	.023
	Heavy Dry	0.5	125	.020
	Light Damp	0.5	100	0.25
	Light Dry	0.2	90	0.20

SAMPLE PROBLEM
The above Kelvin formula can be simplified to:

$$T\text{-}To = (Q' / 2\pi k_s) * I(x)$$

Using the constants as shown in Figure 3 for a Closed Loop/ Open Pipe System.

The sample system is as follows:
Well Diameter = 6”
Inducer Tube = 6 “
Well Water Depth = 550ft
ClimateMaster Heat Pump = W-5-A
Building Heat Loss = 60,000 BTU/Hr
Run Time of 2 days = 48hrs
Assumed Well Ton/Lft = 100 Lft (Q’=120)

Computing x:

$x = r / (2\sqrt{\alpha t})$

$x = (6/12) / (2\sqrt{\alpha t})$
$x = .5 / 2\sqrt{(.05*48)}$
$x = .161$

$\alpha = k / (\rho c_p)$
$\alpha = 2 / (200*.2) = .05$

From integration tables
I(x) = 1.55 (approx.)

Assuming a well design of 100Lft/Ton (heating), the Line Theory Formula then becomes:

$T\text{-}To = (-120 / (2\pi*2.0) * 1.55 = -14.8$ °F

If the ground soil averages 50°F, then the temperature would be depressed to 35.2°F. Just over our desire not to reduce the temperature below 37°F in a well column.

The total heat transferred during two days of continuous running is:

$$q = BTU/Lft * Lft * hrs/week$$
$$q = 120 * 550 *168$$
$$q = 11.09 * 10^6 \text{ BTU's}$$

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HYBRID WELL PERFORMANCE Overflowing of the well by 20% typically increase the k_s, and consequently the α by a factor of 3*. In this example, the well pump would provide 15gom and 12gpm would be returned to the well with 3 (20%) gpm being overflowed to another under-Earth recycle point. The performance of this hybrid system then looks like: x = (6/12) / (2√α*48*3) = .5/(2*8.48) = .29 I(x) = .9904 T-T0 = (120/(2*2.0*3))*.9904 = -3.15°F Considerably less and more desirable temperature reduction in the well water column.	